Maxwell’s equations

Maxwell's Equations:

From electrostatics:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \text{----------------} \quad (1)$$ **Gauss's Law**

$$\vec{\nabla} \times \vec{E} = 0 \qquad \text{----------------} \quad (2)$$

From magnetostatics:

$$\vec{\nabla} . \vec{B} = 0 \qquad \text{--------------} \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad \text{-------------} \quad (4)$$ **Ampere's Law**

$$\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt} \qquad \text{--------------} \quad (5)$$ **Continuity Eqn.**

Modification of Ampere's Law

div of a curl is always zero

From (4) $$\vec{\nabla}\cdot\{\vec{\nabla}\times\vec{B}\} = \vec{\nabla}\cdot\mu_0\vec{J}$$

$$0 = \mu_0(\vec{\nabla}\cdot\vec{J})$$

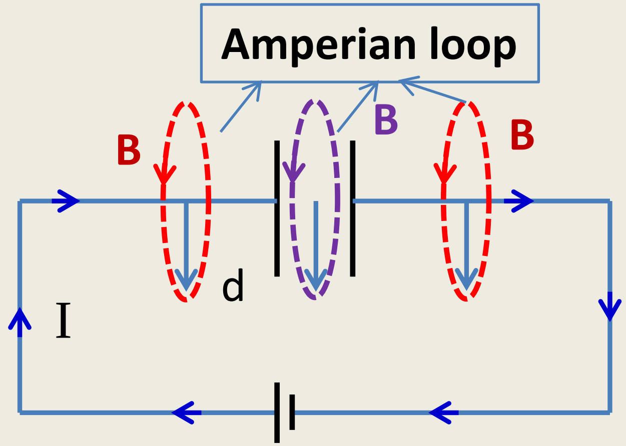
Inconsistency comes since $\vec{\nabla}\cdot\vec{J}$ is NOT always zero

$$\vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}$$ ------------- (5) **Continuity Eqn.**

Ampere's Law cannot be true for general currents which may vary with time

Only for steady currents, it is zero

Failure of Ampere's Law for NON-steady currents



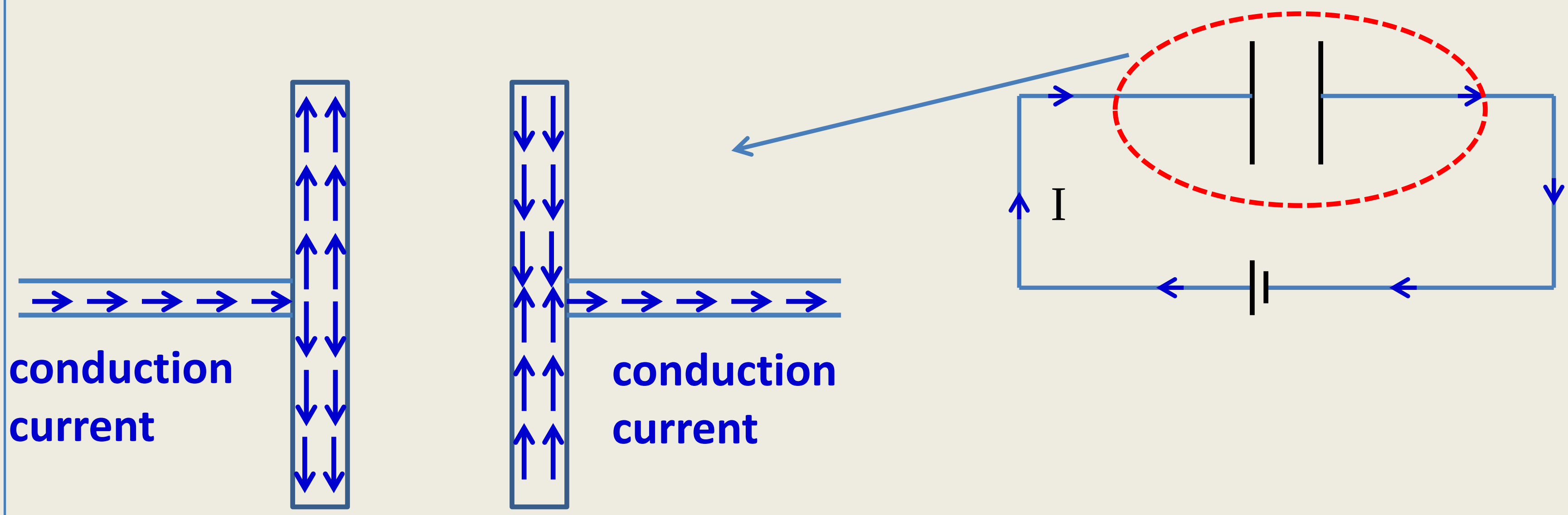
Charging of a capacitor circuit

$$\mathbf{B} = \frac{\mu_0 I}{2\pi d}$$

$\mathbf{B} = 0$ $\qquad I_{encl} = 0$

Magnetic field suddenly disappears, which is NOT physically feasible!

Ampere's Law is not valid if currents are not steady : current enclosed by the loop is an ill-defined notion



How Maxwell fixed Ampere's Law ?

$$\vec{\nabla} \cdot \{\vec{\nabla} \times \vec{B}\} = \mu_0(\vec{\nabla} \cdot \vec{J}) = \mu_0\left(-\frac{d\rho}{dt}\right)$$

$$= -\mu_0 \frac{d}{dt}(\varepsilon_0 \vec{\nabla} \cdot \vec{E}) = -\vec{\nabla} \cdot \left(\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}\right)$$

Add the term on RHS (in brackets) to Ampere's Law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \left(\frac{d\vec{E}}{dt}\right) \longrightarrow$$

Modified Ampere's Law

Ampere's Law with Maxwell term

Then div(curl B) is always zero

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \cdot \vec{J}) + \mu_0 \varepsilon_0 \vec{\nabla} \cdot \left(\frac{d\vec{E}}{dt} \right) = 0$$

$$\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$$

- → **Very small compared to J**
- → **In magnetostatics, E is constant and Ampere's law is valid**
- → **Plays an important role in the propagation of EM wave**
- → **Ordinary EM experiments could not detect this small term**
- → **In 1888, Hertz experiment detected this term**