

Regular Expressions

Dr. Lakshmanan K
SCOPE, VIT, Vellore.

Objective and Outcome

- **Objective**

- To learn what is a regular expression.
- To learn about languages associated with reg. exp.

- **Outcome**

- How to represent regular expressions for a language.
- How to derive the language associated with reg. exp.

Regular expressions

- A **third** way to view regular languages.
- Its like DNA of a language. It's the **pattern** of strings.
- **Definition** of **Regular expressions**:
- The smallest class of strings over $\Sigma \cup \{ (,), *, \cup, \bullet, \phi \}$ that includes:
 1. ϕ and σ for every σ in Σ are reg.exp (**atomic reg. exp.**)
 2. If α and β are reg. exps, so are $\alpha \cup \beta$, $(\alpha)^*$, and $\alpha \bullet \beta$ (i.e., reg. exps. are **closed** under \cup , $*$ and \bullet).
 3. Nothing else is reg. exp. other than obtained by above 1 & 2.

Examples of regular expressions

- **Note:** We drop parentheses and \bullet when not required.
 - Example: $(a \cup b)$ is written $a \cup b$
- Let $\Sigma = \{a, b\}$
- The following are regular expressions:
 - ϕ, a, b
 - $\phi^*, a^*, b^*, ab, a \cup b$
 - $(a \cup b)^*, a^*b^*, (ab)^*$
 - $(a \cup b)^*ab$, etc.
- Which of the following are regular expressions?
 - $\phi \bullet a, *^a, a^*b, ab, (a \cup), (a+b+)$

Language generated by a regular expression r

- **Notation:**
 - $L(r)$ denotes language generated by reg. exp. r .
- $L(\phi) = \phi$
- $L(\sigma) = \{\sigma\}$
- $L(\alpha \cup \beta) = L(\alpha) \cup L(\beta)$
- $L(\alpha \cdot \beta) = L(\alpha) \cdot L(\beta)$
- $L(\alpha^*) = (L(\alpha))^*$

Example Reg.exp $(a + b) \cdot a^*$

$$\begin{aligned}L((a + b) \cdot a^*) &= L((a + b)) L(a^*) \\&= L(a + b) L(a^*) \\&= (L(a) \cup L(b)) (L(a))^* \\&= (\{a\} \cup \{b\}) (\{a\})^* \\&= \{a, b\} \{\lambda, a, aa, aaa, \dots\} \\&= \{a, aa, aaa, \dots, b, ba, baa, \dots\}\end{aligned}$$

More Examples

$$L((a + b \cdot c)^*) = \{\lambda, a, bc, aa, abc, bca, \dots\}$$

$$L((a \cup b)^* a (a \cup b)^*) ?$$

Ans : $\{w \text{ in } \{a, b\}^* \mid w \text{ contains at least one } a\}$

$$r = (aa)^* (bb)^* b$$

$$L(r) = \{a^{2n} b^{2m} b : n, m \geq 0\}$$

Conclusion

- What we learned
 - Regular expressions
 - Examples of regular expression
 - To derive languages from regular expressions
 - Given a (regular) language, the corresponding regular expression of the language.